ON CONNECTEDNESS OF TWO-WAY ELIMINATION OF HETEROGENEITY DESIGNS

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ABSTRACT: The condition under which treatment-row connectedness along with treatment-column connectedness imply doubly-connectedness in case of row-column designs has been developed (Raghavarao et al [1975]). The above result can also be extended to designs involving higher way elimination of heterogeneity.

KEY WORDS: Orthogonal and commutative row-column designs, doubly-connectedness.

Introduction and Preliminaries

In the class of two-way elimination of heterogeneity designs, row-column designs refer to a special class in which the estimates of row effects ignoring treatment classification are orthogonal to the estimates of column effects ignoring treatment classification. Examples of such designs are available in Freeman (1957, 1961), Pearce (1975) and Singh, et al (1978). These designs and all designs available in the present literature are obtained under the condition that each element belonging to the row-column incidence Matrix N* of the design contains unity. In this communication we consider a more general condition on the row-column incidence Matrix N* of the design under which a row-column design exists. In fact, we men-

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ment effect contrast $\mu_1 + \mu_2 = 1$, the commutative row-column design become doubly disconnected with respect to treatment, the contrast being confounded in the whole design.

Theorem 2.2. An orthogonal row-column design is commutative.
Proof:

A row-column design is orthogonal when

$$N^* = \underline{k}\,\underline{p}'/n = N'r^{-\delta}\underset{\sim}{N}. \qquad (1)$$

Under this condition, $M_{01}M_{02} = M_{02}M_{01}$ and the proof is complete.

Theorem 2.3. An orthogonal row-column design which is treatment-row and treatment-column connected is always doubly connected with respect to treatment.

We prove the above in two ways.

First Proof:

Recalling (1), we note that if a particular treatment effect contrast is estimated with loss of information μ_1 from M_{01}, then the same treatment effect contrast is estimated with loss of information μ_2 (=0) from M_{02}. More clearly, for an $\underset{1}{s}^{v\times 1}$ such that $\underset{1}{s}'\underline{r} = 0$, if $M_{01}\,s_1 = \mu_1 s_1$, then $M_{02}\,s_1 = 0s_1$. Hence, recalling $M_{01}\,M_{02} = M_{02}\,M_{01}$, $\mu = \mu_1 + \mu_2 = \mu_1$ $(0 \leqslant \mu_1 < 1)$. Alternatively for an $s_2{}^{v\times 1}$ such that $s_2^1\,{}_2\underline{r} = 0$, if $M_{02}s_2 = \mu_2 s_2$ then $M_{01}s_2 = 0s_2$.

Hence, any treatment effect contrast estimable from each of row and column classification is always estimable from the whole design. Consequently, the design remains doubly connected with respect to treatment.

Alternative Proof: (Generalization of Raghavarao, et al [1975])

For an orthogonal row column design,

$$C = C_1\,r^{-\delta}\,C_2,$$

where $C = r^{\delta} - Nk^{-\delta}N' - \underset{\sim}{N}p^{-\delta}\underset{\sim}{N}' + r\,r'/n,$

$$C_1 = r^{-\delta} - Nk^{\delta}N' ,$$

and $$C_2 = r^{\delta} - \underset{\sim}{N}p^{-\delta}\underset{\sim}{N}'$$

It can be shown that when rank $(C_1) =$ rank $(C_2) = \nu - 1$, then rank $(C) = \nu - 1$ and vice versa.

Thus, we have extended the proof of Raghavarao, et al (1975) under a more general set up.

Illustration

Consider three promising corn hybrids A, B and C arranged according to the following layout in 3 rows and 6 columns.

Row \ Column	1	2	3	4	5	6
1	A	B	B	A	A	B
2	B	C	B	C	C	B
3	A	C	C	A	C	A

The above design is an orthogonal row-column design.

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